2023 Texas A&M REU Miniconference

July 24-25, Blocker Building, Room 306

SCHEDULE

MONDAY, July 24

09:30–09:50 Faith Ellison

Computing Isotopy Types of Zero Sets of Circuit Polynomials

09:55–10:15 Cydnee Evans

Counting and Finding Real Roots of Trinomials

10:20–10:40 Vaishali Miriyagalla

Developing a New Tool for Modeling the Topology of Zero Sets of Near-Circuit Polynomials

10:45–10:55

[Break]

10:55–11:55 Eddie Rohr, Karthik Sellakumaran Latha, Amanda Yin

A Type D Asymmetric Simple Exclusion Process Generated by an Explicit Central Element of $U_q(\mathfrak{so}_{10})$

TUESDAY, July 25

09:30–09:50 Michaela Thompson

Identifiability of Linear Compartmental Model Parameter Subsets

09:55–10:15 Georgia Corbett

Extremal values of newform Dedekind sums

10:20–10:40 Katherine Clemens

Analyzing Eleven Bistable Gene Regulatory Networks for Hopf Bifurcations

10:45–10:55

[Break]

10:55–11:15 Jonathan Martinez

Understanding the Properties, Operations, and Identifiability Degree of One-Input/One-Output Mammillary Models

11:20–12:00 Elena De Leon and Wade McCormick

The Image of Newform Dedekind Sums attached to Quadratic Characters

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ABSTRACTS

(in order of appearance)

Computing Isotopy Types of Zero Sets of Circuit Polynomials

(Faith Ellison)

Understanding real curves is more subtle than understanding complex curves, as one can see from the literature on Hilbert's 16th Problem. This includes the determination of isotopy types, which means counting ovals and determining their possible dispositions.

Newton polygons and amoebae are two tools that arose from studying complex curves and we'll see ways to refine these tools to tell us about real curves. In particular, we'll see an efficient algorithm to determine the isotopy type of the positive zero set of any bivariate tetranomial, including the high degree case. As a consequence, we'll see how it's better to use explicit quadratic forms instead of Viro diagrams to determine isotopy type.

Counting and Finding Real Roots of Trinomials

(Cydnee Evans)

The fastest algorithms for approximating the real roots of degree d polynomials have complexity near linear in d, but still do not run in time polynomial in the input size. For polynomials with few terms, the input size can be linear in log d, and binomials can be solved over the real numbers in time quadratic in log d. The remaining open case is univariate trinomials, and it was proved around 2009 that one can at least count real roots for trinomials in time quadratic in log d, using Baker's Theorem on linear forms in logarithms. As for approximating real roots, a promising approach involves A-hypergeometric series. However, the case of trinomials that “almost” have a multiple root has proved difficult. We report on recent progress involving a new family of hypergeometric series.

Developing a New Tool for Modeling the Topology of Zero Sets of Near-Circuit Polynomials

(Vaishali Miriyagalla)

There are many practical applications for the zero sets of polynomials, but they are time-consuming to solve, considered to by NP hard. A-discriminants help group sets of coefficients with zero sets of the same topology. For near circuits (polynomials with n+3 terms, where n is the number of variables), the A-discriminant can be reduced to two dimensions, making this grouping easier. This presentation will focus on the implementation of a program that can help model the isotopy types of zero sets of bivariate pentanomials.

A Type D Asymmetric Simple Exclusion Process Generated by an Explicit Central Element of $U_q(\mathfrak{so}_{10})$

(Eddie Rohr, Karthik Sellakumaran Latha, Amanda Yin)

A \textbf{type D asymmetric simple exclusion process (type D ASEP)} is a two-species asymmetric interacting particle system where a site may contain two particles of different species. Previous research in 2020 by Kuan, Landry, Lin, Park, and Zhou has found that a type D ASEP can be generated using a central element from the type D quantum groups $U_q(\mathfrak{so}_6)$ and $U_q(\mathfrak{so}_8)$. The authors conjectured that this process can be generalized to $U_q(\mathfrak{so}_{2n})$. We examine this claim in the case of $U_q(\mathfrak{so}_{10})$. We first construct a central element of this quantum group using general methods outlined by Kuan. Specifically, we complete a long algebraic calculation aided by a computer. To reduce the computation time to a tractable amount we adapt and optimize algorithms from the work in 2020. Then, we construct a type D ASEP with the corresponding quantum Hamiltonian.

Identifiability of Linear Compartmental Model Parameter Subsets

(Michaela Thompson)

This presentation will discuss research done on how linear compartmental model parameter subset identifiability is affected by adding and removing different components of the models, such as leaks, inputs, and outputs. Some past research has found conclusions regarding how adding and removing these components will affect an identifiable model, but not in the case where the model is unidentifiable. The research presented here examines this case. In

particular, we will look at cycle models where some, but not all of the parameters, can be recovered from the model's input-output equation, determine how this subset of identifiable parameters changes as a result of adding and removing components, and explore what the model's input-output equations tell us about why these changes occur.

Extremal values of newform Dedekind sums

(Georgia Corbett)

We study a generalized Dedekind sum $S_{\chi_1,\chi_2}(a,c)$ attached to the weight zero newform Eisenstein series $E_{\chi_1,\chi_2}(z,s)$. Our work shows the newform Dedekind sum is small for almost all values using properties of continued fractions. Further, we calculate examples of large values attained by the Dedekind sum, thereby showing a nontrivial bound cannot be found.

Analyzing Eleven Bistable Gene Regulatory Networks for Hopf Bifurcations

(Katherine Clemens)

Gene regulatory networks, also referred to as GRNs, consist of sets of genes that all interact to perform many important cellular functions, including cellular differentiation and cell cycle control. By cataloguing the behaviors of these networks under various initial conditions, we learn more about how these networks function and, therefore, gain some insight into cellular functions as well. The focus of this paper is the behavioral analysis of 11 small bistable GRNs that all interact to form over 40,000 important larger biochemical networks. To be more specific, we searched for the presence of Hopf bifurcations in all 11 networks, as well as the location and parameter values required for this behavior to be expressed. Of the eleven networks in question, we verified that 4 do not have Hopf bifurcations, while the remaining 7 may still exhibit this behavior. The Routh-Hurwitz stability criterion was used to exclude the possibility of Hopf bifurcations in certain networks. If this criterion did not exclude the network, the location and parameter values of the possible Hopf bifurcation will be calculated using other methods.

Understanding the Properties, Operations, and Identifiability Degree of One-Input/One-Output Mammillary Models

(Jonathan Martinez)

This work discusses the unique properties of a type of Linear-Compartmental Model called a One-Input/One-Output Mammillary Model. Although this model has several applications in pharmacological, ecology, and other biological settings, previous research has only determined one property: the structural identifiability (if parameters of the model can be recovered from data). We went one step further and determined if each of the individual parameter values were globally identifiable (there is only one possible value the parameter can take), locally identifiable (there is a finite number of values the parameter can take), or unidentifiable (there are infinitely many solutions that the parameter can take). This was done by using an application called SIAN (Structural Identifiability ANalyser) whose results were then

proven by analyzing the coefficients of the model's Input-Output Equation (a type of equation that can be derived from Linear-Compartmental Models).

The Image of Newform Dedekind Sums attached to Quadratic Characters

(Elena De Leon and Wade McCormick)

We study the image of the newform Dedekind sum attached to the weight zero newform Eisenstein series $E_{\chi_1,\chi_2}(z,s)$, in the case in which χ_1 and χ_2 are quadratic. We put forward a number of characterizations of the image in particular cases, and demonstrate a closed-form formula for class of such characters and values.