Special Topics. Metric Invariants: algorithmic and geometric applications.

MATH 689, FALL 2020

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Meeting Times-Location

Lectures: TBA — Classroom: TBA

Office hours: by appointment

Global Course Description

Name: Special Topics. Metric Invariants: algorithmic and geometric applications.

Webpage: `http://www.math.tamu.edu/~florent/teaching/689Fall20.html`

Description: We will study sophisticated and powerful metric invariants and their connections with modern cutting-edge techniques in the design of approximation algorithms for optimization problems (e.g. Sparsest Cut, Nearest Neighbor Search), in geometric group theory (compression theory), and in Alexandrov geometry (coarse universality). In particular we will showcase interesting applications of harmonic analysis, martingales, random walks on graphs and Markov chains in general, to the theory of metric invariants and their applications to pure mathematics and theoretical computer science.

Textbook(s): Matousek's lecture notes on metric embeddings (freely available online)

Prerequisities: The course will be designed to be as inclusive as possible to most of the science and engineering majors and is ultimately subject to the instructor's approval. Students majoring CSCE, ECEN, ISEN, STAT, are strongly encourage to enroll and to express their interest directly to the instructor. For instance NO programming background is required and NO particular mathematics class is a prerequisite. However, it is expected that students wishing to enroll in this class are familiar with:

- basics of linear algebra (as can be found in MATH 304, 309, 311, 323)
- basics of discrete probability theory (e.g. STAT 211, ECEN 303, or MATH 411)

Familiarity in proof writing (e.g. MATH 300) and discrete mathematics (as can be found in MATH 302 or CSCE 222/ECEN 222) is not strictly necessary but will be beneficial.

Learning Objectives: Our ability to faithfully represent an a priori unstructured geometric object into a highly structured one has proven to be one of the most powerful lock breaker to some of the hardest problems, whether in pure mathematics or theoretical computer science. One of the main goal of this course is to give a thorough exposure to numerous techniques and ideas that rely on this geometric approach and that are fundamental in data science and in pure mathematics. We expect the course to be of interest to students from various majors and scientific backgrounds. One objective of this course will be to foster interactions between the students. The very active and popular field of research to be presented in this course is rapidly evolving. Numerous important open problems will be discussed with the ultimate goal to trigger the interest of starting graduate students (and senior undergraduate students) into pursuing research work in this direction.

Course content: We now describe a list of topics we will choose from to build the final course content.

Basics of metric geometry and embedding theory
- metric and normed spaces, Hilbert and Lebesgue spaces, graphs and groups as geometric objects,
- bi-Lipschitz embeddings and distortion
- coarse embeddings and compression
- Bourgain's embedding technique
- Johnson-Lindenstrauss dimension reduction technique

Algorithmic problems
- Sparsest Cut
- Approximate Nearest Neighbor Search
- Computing the ℓ_p-distortion
- Flows and cuts sparsifiers
- Error correcting codes
- Compressive Sensing

Ribe program and metric invariants
- Ribe's rigidity theorem
- Rademacher type and Rademacher cotype
- uniform convexity and uniform smoothness
- Metric invariants

Metric types and applications
- Enflo type and the geometry of the Hamming cubes
- Pisier's inequality and Enflo's problem
- Bourgain-Milman-Wolfson type
- Enflo type and UMD Banach spaces
- Ball's Markov type and the Lipschitz extension problem
- flow and cut sparsifiers and the Lipschitz extension problem
- Markov type and Guentner-Kaminker compression exponent theory
- Markov type of planar graphs
- Markov type and graphs with large girth

Metric cotypes and applications
- Mendel-Naor metric cotype and the geometry of ℓ_∞-grids
- metric cotype of barycentric metric spaces
- nonlinear martingales and Pisier's martingale cotype inequality in barycentric metric spaces
- Eskenazis-Mendel-Naor solution of Gromov's problem about coarse universality of Alexandrov spaces of nonpositive curvature
- metric Markov cotype and nonlinear spectral gaps
- construction of expander graphs based on zig-zag products

Markov convexity and applications
- Markov convexity and the geometry of trees, diamond graphs, and Laakso graphs
- martingale cotype and Pisier's renorming theorem
- tree dichotomy
- Markov convexity of Alexandrov spaces of nonnegative curvature
- polynomial time approximation algorithm to compute the ℓ_p-distortion of tree metrics

Tentative Schedule:

Week 1: Basics of metric geometry and embedding theory
Week 2-3: Algorithmic problems (e.g. computing the ℓ_p-distortion, Sparsest Cut and Nearest Neighbor Search)
Week 3-5: Ribe Program and metric invariants
Week 5-8: Metric types and applications
Week 9-11: Metric cotype and applications
Week 12-13: Markov convexity and applications
Week 14: Thanksgiving Holiday
Week 15: Markov convexity and applications

Grading Policy

Collaborative projects: The collaborative projects are an integral part of the learning experience of this course. On a bi-weekly basis you will be assigned a project which will have two parts:

- a first part which will typically consist of a few homework problems for which you will be asked to provide a personal solution. The grading of this part will follow classical grading rules for traditional homework.
- a second part which is not of a traditional homework style but rather in the spirit of a (very modest) REU project. You will be provided with a problem, mostly of a computational nature, related to the current topic being covered. In collaboration with the other members of you team (details about forming the teams will be discussed during the first week of class) you will elaborate a strategy to solve the problem and show and explain your findings, successes and failures alike, in writing (one write up per team). For this part, it will be your critical thinking that will be evaluated.

Participation: You will receive a grade for your involvement in the class. For examples (but not limited to) high attendance rate, showing leadership and initiative in the collaborative projects, volunteering to scribe a lecture, will be evaluated positively.

Final Grade: The final grade will be computed as follows: final grade = collaborative projects 80%+ participation 20%.

Grading: A 90-100% B 80-89% C 70-79% D 60-69% F 0-59%

Appeal: Due to FERPA privacy issues, I cannot discuss grades over email or phone. If you have a question about your grade, please come see me in person.

Attendance Policy

- Excused absences: The University views class attendance as an individual student responsibility. It is essential that students attend class and complete all assignments to succeed in the course. University student rules concerning excused and unexcused absences can be found at http://student-rules.tamu.edu/rule07. In particular, late assignments will NOT be allowed unless a University approved reason is given to me in writing. Notification before the absence is required when possible. I will not accept the "University Explanatory Statement for Absence from Class" form. Further, an absence due to a non-acute medical service or appointment (such as a regular checkup) is not an excused absence. Providing a fake or falsified doctor's note or other falsified documentation is considered academic dishonesty, will be reported to the Aggie Honor Council, and will result in an F in the course.

Miscellaneous

Academic Integrity Statement

Cheating and other forms of academic dishonesty will not be tolerated.

Aggie Honor Code: "An Aggie does not lie, cheat, or steal, or tolerate those who do"

Upon accepting admission to Texas A&M University, a student immediately assumes a commitment to uphold the Honor Code, to accept responsibility for learning, and to follow the philosophy and rules of the Honor System. Students will be required to state their commitment on examinations, research papers, and other academic work. Ignorance of the rules does not exclude any member of the TAMU community from the requirements or the processes of the Honor System. For additional information please visit: `http://aggiehonor.tamu.edu`

Title IX and Statement on Limits to Confidentiality

Texas A&M University and the College of Science are committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws provide guidance for achieving such an environment. Although class materials are generally considered confidential pursuant to student record policies and laws, University employees — including instructors — cannot maintain confidentiality when it conflicts with their responsibility to report certain issues that jeopardize the health and safety of our community. As the instructor, I must report (per Texas A&M System Regulation 08.01.01) the following information to other University offices if you share it with me, even if you do not want the disclosed information to be shared: • Allegations of sexual assault, sexual discrimination, or sexual harassment when they involve TAMU students, faculty, or staff, or third parties visiting campus. These reports may trigger contact from a campus official who will want to talk with you about the incident that you have shared. In many cases, it will be your decision whether or not you wish to speak with that individual. If you would like to talk about these events in a more confidential setting, you are encouraged to make an appointment with the Student Counseling Service (https://scs.tamu.edu/). Students and faculty can report non-emergency behavior that causes them to be concerned at http://tellsomebody.tamu.edu.

Disability Services

The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, please contact Disability Services, currently located in the Disability Services building at the Student Services at White Creek complex on west campus or call 979-845-1637. For additional information, visit http://disability.tamu.edu.

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