Course Information

Course Number: Math 689
Course Title: Nonlinear Waves and Dispersive Equations
Section: TBD
Time: TBD
Location: TBD
Credit Hours: 3

Instructor Details

Instructor: Jonas Luhrmann
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Office Hours: TBD

Course Description

Nonlinear dispersive and hyperbolic equations describe a variety of wave phenomena in nature: from the dynamics of Bose-Einstein condensates and the propagation of laser beams to the spreading of gravitational waves. This course is an introduction to the study of such nonlinear partial differential equations with a focus on understanding the local existence, the global existence, and the asymptotic behavior of solutions.
We will begin by assembling several frequently used tools from harmonic analysis and functional analysis. In the first major part of the course we will develop the so-called vector field method to investigate the long-time behavior of solutions to nonlinear hyperbolic PDEs. In the second part of the course we will study the nonlinear Schrödinger equation, which is a prototype of a dispersive equation. Here we will learn how to use conservation laws and monotonicity formulas to prove global existence and scattering of solutions. If time permits, we will conclude the course with an outlook on some current research topics revolving around long-range scattering and the asymptotic stability of solitons.

Course Prerequisites

First-year graduate analysis sequence (Math 607/608) or equivalent or instructor's approval. Some familiarity with partial differential equations (such as Math 412 or Math 611) would be helpful, but is not necessary.

Special Course Designation

None

Course Learning Outcomes

The students will be able to use vector field and harmonic analysis techniques to study the local and global existence of solutions to nonlinear dispersive and hyperbolic equations.

Textbook and/or Resource Materials

Lecture notes will be provided. References to textbooks and other lecture notes will be posted on the course webpage when relevant.

Grading Policy

There will be biweekly homework assignments that will be graded by the instructor.

The final course grade will be determined from the overall homework score according to the following grading scale: 85%-100% = A, 70%-84% = B, 60%-69% = C, 50%-59% = D, <50% = F.

Late Work Policy

I will accept homework that is up to one day late.

Course Schedule

I. Introduction and Motivation (1 week)

II. Mathematical Tool Box (2 weeks)
Fourier transform, Sobolev spaces, stationary phase, Gronwall's lemma, bootstrapping

III. Nonlinear Hyperbolic Equations (7 weeks)
Linear wave equation via fundamental solution and representation formula, linear wave equation via Fourier transform, energy estimates, local existence theory for nonlinear wave equtions, decay of solutions via energy methods, small data global regularity, null condition

IV. Nonlinear Dispersive Equations (4 weeks)
Dispersion, the Schrödinger equation as a prototype of a dispersive equation, decay estimates and Strichartz estimates, local existence for semilinear Schrödinger equations, conservation laws, monotonicity formulas, global existence and scattering for defocusing energy sub-scritical Schrödinger equations

V. Outlook (1 week)

University Policies

Attendance Policy

The university views class attendance and participation as an individual student responsibility. Students are expected to attend class and to complete all assignments.

Please refer to Student Rule 7 in its entirety for information about excused absences, including definitions, and related documentation and timelines.

Makeup Work Policy

Students will be excused from attending class on the day of a graded activity or when attendance contributes to a student's grade, for the reasons stated in Student Rule 7, or other reason deemed appropriate by the instructor.

Please refer to Student Rule 7 in its entirety for information about makeup work, including definitions, and related documentation and timelines.

Absences related to Title IX of the Education Amendments of 1972 may necessitate a period of more than 30 days for make-up work, and the timeframe for make-up work should be agreed upon by the student and instructor" (Student Rule 7, Section 7.4.1).

"The instructor is under no obligation to provide an opportunity for the student to make up work missed because of an unexcused absence" (Student Rule 7, Section 7.4.2).

Students who request an excused absence are expected to uphold the Aggie Honor Code and Student Conduct Code. (See Student Rule 24.)

Academic Integrity Statement and Policy

"An Aggie does not lie, cheat or steal, or tolerate those who do."

"Texas A&M University students are responsible for authenticating all work submitted to an instructor. If asked, students must be able to produce proof that the item submitted is indeed the work of that student. Students must keep appropriate records at all times. The inability to authenticate one's work, should the instructor request it, may be sufficient grounds to initiate an academic misconduct case" (Section 20.1.2.3, Student Rule 20).

You can learn more about the Aggie Honor System Office Rules and Procedures, academic integrity, and your rights and responsibilities at aggiehonor.tamu.edu.

Americans with Disabilities Act (ADA) Policy

Texas A&M University is committed to providing equitable access to learning opportunities for all students. If you experience barriers to your education due to a disability or think you may have a disability, please contact Disability Resources in the Student Services Building or at (979) 845-1637 or visit disability.tamu.edu. Disabilities may include, but are not limited to attentional, learning, mental health, sensory, physical, or chronic health conditions. All students are encouraged to discuss their disability related needs with Disability Resources and their instructors as soon as possible.

Title IX and Statement on Limits to Confidentiality

Texas A&M University is committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws prohibit gender-based discrimination and sexual harassment, including sexual assault, sexual exploitation, domestic violence, dating violence, and stalking.

With the exception of some medical and mental health providers, all university employees (including full and part-time faculty, staff, paid graduate assistants, student workers, etc.) are Mandatory Reporters and must report to the Title IX Office if the employee experiences, observes, or becomes aware of an incident that meets the following conditions (see University Rule 08.01.01.M1):

- The incident is reasonably believed to be discrimination or harassment.
- The incident is alleged to have been committed by or against a person who, at the time of the incident, was (1) a student enrolled at the University or (2) an employee of the University.

Mandatory Reporters must file a report regardless of how the information comes to their attention – including but not limited to face-to-face conversations, a written class assignment or paper, class discussion, email, text, or social media post. Although Mandatory Reporters must file a report, in most instances, you will be able to control how the report is handled, including whether or not to pursue a formal investigation. The University's goal is to make sure you are aware of the range of options available to you and to ensure access to the resources you need.

Students wishing to discuss concerns in a confidential setting are encouraged to make an appointment with Counseling and Psychological Services (CAPS).

Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the University's Title IX webpage.

Statement on Mental Health and Wellness

Texas A&M University recognizes that mental health and wellness are critical factors that influence a student's academic success and overall well-being. Students are encouraged to engage in proper self-care by utilizing the resources and services available from Counseling & Psychological Services (CAPS). Students who need someone to talk to can call the TAMU Helpline (979-845-2700) from 4:00 p.m. to

8:00 a.m. weekdays and 24 hours on weekends. 24-hour emergency help is also available through the National Suicide Prevention Hotline (800-273-8255) or at suicidepreventionlifeline.org.

COVID-19 Temporary Amendment to Minimum Syllabus Requirements

The Faculty Senate temporarily added the following statements to the minimum syllabus requirements in Fall 2020 as part of the university's COVID-19 response.

Campus Safety Measures

To promote public safety and protect students, faculty, and staff during the coronavirus pandemic, Texas A&M University has adopted policies and practices for the Fall 2020 academic term to limit virus transmission. Students must observe the following practices while participating in face-to-face courses and course-related activities (office hours, help sessions, transitioning to and between classes, study spaces, academic services, etc.):

- Self-monitoring—Students should follow CDC recommendations for self-monitoring. **Students who have a fever or exhibit symptoms of COVID-19 should participate in class remotely and should not participate in face-to-face instruction.**
- Face Coverings—Face coverings (cloth face covering, surgical mask, etc.) must be properly worn in all non-private spaces including classrooms, teaching laboratories, common spaces such as lobbies and hallways, public study spaces, libraries, academic resource and support offices, and outdoor spaces where 6 feet of physical distancing is difficult to reliably maintain. Description of face coverings and additional guidance are provided in the Face Covering policy and Frequently Asked Questions (FAQ) available on the Provost website.
- Physical Distancing—Physical distancing must be maintained between students, instructors, and others in course and course-related activities.
- Classroom Ingress/Egress—Students must follow marked pathways for entering and exiting classrooms and other teaching spaces. Leave classrooms promptly after course activities have concluded. Do not congregate in hallways and maintain 6-foot physical distancing when waiting to enter classrooms and other instructional spaces.
- To attend a face-to-face class, students must wear a face covering (or a face shield if they have an exemption letter). If a student refuses to wear a face covering, the instructor should ask the student to leave and join the class remotely. If the student does not leave the class, the faculty member should report that student to the Student Conduct office for sanctions. Additionally, the faculty member may choose to teach that day's class remotely for all students.

Personal Illness and Quarantine

Students required to quarantine must participate in courses and course-related activities remotely and **must not attend face-to-face course activities**. Students should notify their instructors of the quarantine requirement. Students under quarantine are expected to participate in courses and complete graded work unless they have symptoms that are too severe to participate in course activities.

Students experiencing personal injury or Illness that is too severe for the student to attend class qualify for an excused absence (See Student Rule 7, Section 7.2.2.) To receive an excused absence, students must comply with the documentation and notification guidelines outlined in Student Rule 7. While Student Rule 7, Section 7.3.2.1, indicates a medical confirmation note from the student's medical provider is preferred, **for Fall 2020 only, students may use the Explanatory Statement for Absence from Class form in lieu of a medical confirmation. Students must submit the Explanatory Statement for Absence from Class within two business days after the last date of absence.**