Title: Introduction to Probability in Banach Spaces and Applications

Course Description:
This course will cover fundamental probabilistic inequalities/notions that should be of interest to a wide range of students
(and that every student in Analysis should know about) in a somewhat organized fashion.

Some real-valued inequalities:
- Khintchine's inequalities (applications to isomorphic embedding of \ell_2 into L_p)
- Rosenthal's inequalities
- Johnson-Maurey-Schechtman-Tzafriri inequalities (application to quantitative obstruction to isomorphic embeddability between L_p-spaces) stable random variables (application to isometric embeddings between L_p-spaces)
- Concentration of Lipschitz functions (application to dimension reduction via Johnson-Lindenstrauss lemma)

Some vector-valued inequalities:
- Basics of Banach space-valued random variables
- Symmetrization
- Kahane's contraction principle
- Khintchine-Kahane inequalities

Type and cotype:
- Basic theory and duality
- Type and cotype of classical Lebesgue spaces (application to quantitative obstruction to isomorphic embeddings)
- Pisier's \ell_1-theorem

Some martingale inequalities:
- Pisier's martingale type and martingale cotype (application to uniformly convex/smooth renormings)
- Martingale inequalities in barycentric metric spaces
- Application to metric invariants in the Ribe program (e.g., Markov type, Enflo type, Markov convexity....) and the geometry of graphs (e.g., with large girth, Hamming cubes, trees, diamonds....)
- No dimension reduction in \ell_1

Pisier's inequality:
- Basics of Harmonic Analysis on the discrete hypercubes
- Basics of Markov semigroups (mostly the case of the heat semigroup on the discrete hypercubes)
- Application of Pisier's inequality to the Enflo problem
- Pisier's inequality in UMD spaces

Talagrand's influence inequality:
- The classical Poincaré inequality
- Bonami-Beckner hypercontractivity inequality

Derivation of the KKL-inequality from Talagrand's influence inequality
Applications to Analysis of Boolean Functions and Social Choice.
KKL-type is equivalent to type 2

If time permits:
Vector-valued versions of Talagrand's influence inequality
Talagrand type and embeddability of quotients of the Hamming cubes