AB Exam

Texas A&M High School Math Contest

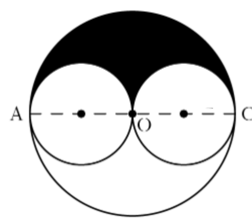
2 November, 2024

(NOTE: If units are appropriate, please include them in your answer. All answers must be simplified where possible.)

1. All positive integers are written consecutively (starting from 1) as a single sequence of decimal digits. Find the 2024th digit in that sequence.

2. If you walk for 45 minutes at a rate of 4 mph and then run for 30 minutes at a rate of 10mph, what is your average speed over the entire trip, in miles per hour?

3. In the figure below, circle O has diameter $\overline{AC}$, and the smaller circles have diameters $\overline{AO}$ and $\overline{OC}$. If the radii of the smaller circles is 24, what is the shaded area?



4. In a soccer tournament, every two teams played each other twice. What was the number of participating teams if the total number of games played was 182?

5. The numbers -2, 11, 2, 20, and 24 are arranged according to the rules below:

   - The largest isn't first, but it is in one of the first three places.
   - The smallest isn't last, but it is in one of the last three places.
   - The median isn't first or last.

   What is the average of the first and last numbers?

6. $(2024)^2 = 4,096,576$. From this number, subtract $(2023)(2024)$, then add $(2022)(2024)$. Continue this pattern of alternating subtraction and addition all the way down to $(1)(2024)$. What is the total?

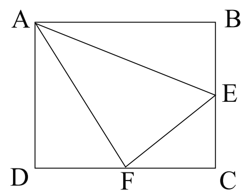
7. Find the minimal possible value of the expression $x + \dfrac{2}{x}$, where $x > 0$.

8. Solve for x: $3^x + 3^x + 3^x = 9^{2x}$.

9. Find a solution (x, y, z) of the following system of equations:

$$\begin{cases} 2x + 3y + 2z = 3, \\ 3x + 2y + 2z = 5, \\ 2x + 2y + 3z = 6. \end{cases}$$

10. Suppose the area of rectangle $ABCD$ is 224, point E is the midpoint of $\overline{BC}$, and point F is the midpoint of $\overline{CD}$ (see figure below). What is the area of $\triangle AEF$?



11. The sum of 24 consecutive odd numbers is 12,000. What is the largest number?

12. Find a positive number x such that $x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{x}}}$.

13. A six-digit integer is written down. If the units digit is moved to the front of the number (for example, 123456 becomes 612345), the new integer is five times the original integer. What is the original integer?

14. Ten thousand light bulbs, numbered 1 to 10000, are all initially turned on. Each bulb has a switch that toggles its state (turning it on if it's off, or off if it's on). Person 1 toggles the switch on bulb #2 (the first prime number), and every multiple of 2. Person 2 toggles the switch on bulb #3 (the second prime number) and every multiple of 3. Person 3 toggles the switch on bulb #5 (the third prime number) and every multiple of 5. The pattern continues, with Person N toggling the switch on the bulb numbered with the Nth prime number and every multiple of that prime number. After all the people have completed their turns, which of the following bulbs will be on? If none, write **NONE**:

24, 112, 2024, 2025

15. Let c be a real solution of the equation $x^4 - 3x + 1 = 0$. Evaluate the expression $c^6 + c^4 - 3c^3 + c^2 - 3c$.

16. Consider a fraction $\dfrac{6n-1}{7n+1}$, where n is a positive integer. Find the smallest value of n for which the fraction is not in lowest terms.

17. How many integer solutions (x, y) does the following system of inequalities have?

$$\begin{cases} 2x + y < 8, \\ 3x - 4y < 1, \\ x > 0. \end{cases}$$

18. Find a real solution of the equation $\sqrt{x-1} + \sqrt{x-3} = 2$.

19. All real solutions of the inequality $\sqrt{3 - 2x - x^2} > x + 1$ fill an interval of the real line. Find the length of that interval.

20. How many distinct real roots does the following equation have:

$$(2x^2 - 5x + 2)^3 + (6x^2 - x - 1)^3 = (8x^2 - 6x + 1)^3\,?$$