# DE Exam

**Texas A&M Math Contest**

2 November, 2024

(NOTE: If units are appropriate, please include them in your answer. All answers must be simplified where possible.)

1. Let $X = \log(2000) + \log(2001) + \log(2002) + \cdots + \log(2024)$ and let $Y = \log(100/2000) + \log(100/2001) + \log(100/2002) + \cdots + \log(100/2024)$. Compute $X + Y$.

2. Evaluate $1 - 2 + 3 - 4 + 5 - 6 + \cdots + 2023 - 2024$.

3. The function $f$ has the property that, for each real number $x$ in its domain,

$$f(x) + f\left(\frac{1}{x}\right) = x$$

   What is the largest possible domain of $f$?

4. Find a real number $b$ such that the equation $|x + 20| + |x + 3| = b$ has infinitely many solutions.

5. What is the smallest positive integer $n$ such that $n$ is divisible by 24, $n^2$ is a perfect cube, and $n^3$ is a perfect square? You may leave your answer in the form of an exponent.

6. Suppose that $(3^a)^b = 3^a \times 3^b$. If $b = 5$, find $a$.

7. Given $\tan(\theta°) = \dfrac{\cos(24°) - \sin(24°)}{\cos(24°) + \sin(24°)}$, what is the smallest positive value of $\theta$?

8. Solve the equation

$$\log_3(x) + \log_3(x^2 - 3x + 3) = 0$$

9. For any real number $x$, we have $3f(x) + f(8 - x) = x^3$. Find $f(4)$

10. Given the circles centered at the origin with radius 1 and 2. Create square $ABCD$ such that:

- $\overline{CD}$ is tangent to the smaller circle at its midpoint $(0, 1)$, and
- points $A$ and $B$ lie on the larger circle above $\overline{CD}$.

What is the side length of the square?

11. Ten thousand light bulbs, numbered 1 to 10000, are all initially turned on. Each bulb has a switch that toggles its state (turning it on if it's off, or off if it's on). Person 1 toggles the switch on bulb #2 (the first prime number), and every multiple of 2. Person 2 toggles the switch on bulb #3 (the second prime number) and every multiple of 3. Person 3 toggles the switch on bulb #5 (the third prime number) and every multiple of 5. The pattern continues, with Person $N$ toggling the switch on the bulb numbered with the $N$th prime number and every multiple of that prime number. After all the people have completed their turns, which of the following bulbs will be on? If none, write **NONE**:

24, 112, 2024, 2025

12. Let $p$ be the greatest prime factor of 9991. Compute the sum of the digits of $p$.

13. Let $A$, $B$, and $C$ be three distinct points on the graph of $y = x^2$, with $\overline{AB}$ parallel to the $x$-axis and $\triangle ABC$ a right triangle with area 2024. What is the sum of the digits of the $y$-coordinate of $C$?

14. In the figures below, each tiny square is one square unit of area (so the area of Figure 0 is 1, and so on). If we continue to follow the pattern, what is the area of Figure 200?

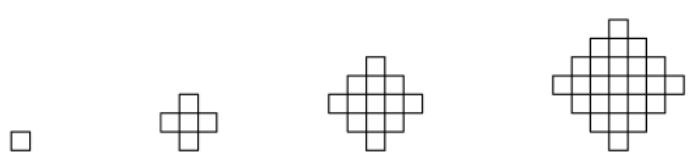

Figure 0 Figure 1 Figure 2 Figure 3

15. Let $a$, $b$, $c$ and $d$ be real numbers. Let $P(x) = ax^9 + bx^5 + cx^3 + dx + 8$. Suppose $P(-3) = 9$. Find $P(3)$.

16. Given $1001 = abc + ab + ac + bc + a + b + c + 1$, where $a, b, c$ are positive integers. Compute $abc$.

17. Find the number of real ordered pairs $(a,\ b)$ which are solutions to the complex equation $(a + bi)^{2024} = a - bi$.

18. Let $c$, $d$, and $e$ be real numbers such that the polynomial $x^4 - 24x^3 + cx^2 + dx + e$ has four distinct roots which are positive integers in arithmetic progression. What is $e$?

19. The square below is a multiplicative magic square, where the product of the numbers in each row, column, and diagonal is the same. If all the entries are positive integers (not necessarily distinct), how many different possible values are there for $h$?

| | | |
|---|---|---|
| 6 | $b$ | $c$ |
| $d$ | $e$ | $f$ |
| $g$ | $h$ | 24 |

20. In $\triangle BAC$, $\angle BAC = 40^\circ$, $AB = 2$, and $AC = 4$. Points $D$ and $E$ lie on $\overline{AB}$ and $\overline{AC}$ respectively. If the minimum possible value of $BE + DE + CD = \sqrt{N}$, what is $N$?