2024 Power Team
Texas A&M High School Students Contest
2 November 2024

- Each power team entry must have a cover sheet (**typed**). The cover sheet must contain the full name of the school (don't abbreviate), the coach's name, contact email, physical address (so that we know where to send the prizes in case your team wins), the team's name, and the names of each team member. For example, if a school has 3 power team entries, then there should be three different team names. The following is an example of an acceptable title page:

  2024 Power Team Entry
  XXXX School, Team 1
  Coach: Ms. Wizard
  Contact email:
  Address:
  Team Members:
  Jane Doe
  John Smith
  etc.

- Team participants are not allowed to consult with anyone but their teammates. Participants are not allowed to look on the web (including artificial intelligence tools) for any information regarding the power team exam, nor are they allowed to search books or other reference materials.
- Team entries are expected to be neat and legible. If not, they face the possibility of being disqualified by the judges.
- It is expected that you show all work, clearly presenting all steps and/or calculations done that lead to the final answer/conclusion.
- **Submissions:**
  - **Hand delivered** submissions are due by 9:00 am on the day of the contest. We will have two boxes for collecting these exams (one on the registration table and another one in the coaches breakroom).
  - **Online submission** should be done no later than 8:30 AM on the day of the contest (Saturday, November 2). Send your solution to our new email **math-hsmc@tamu.edu** (you can equally use the previous email **hsmc@math.tamu.edu**) and make your school's and team's names as your email's subject line. Make sure the solution file is in PDF format (if your solution is handwritten, scan it first and save it as a **single** PDF file).

In case you need assistance with the exam submission, contact David Manuel (email: **dmanuel@tamu.edu**).

GOOD LUCK!

Stopping the flood

A landscape in the Square Country is an infinite checkerboard: each cell is Water, Ground, or Rock.

If the Ground cell is adjacent by side to a Water cell on Day n, then this cell will be flooded overnight, i.e. it will be a Water cell on Day $n+1$. Rock cells stay Rock, and Water cells stay Water.

FIGURE 1. Left: landscape on Day 1. Water is gray, Ground is white, Rock is black. Right: same landscape on Day 2.

(1) On Day 1, there is only one Water cell and no Rock cells. Find the formula for the number of Water cells on Day n.
(2) Suppose there are no Rock cells. Which configurations are possible Water configurations on Day 2? Describe a criterion to distinguish possible configurations from impossible configurations.

FIGURE 2. Left: possible configuration on Day 2. Right: impossible configuration on Day 2.

(3) We will say that *the flood has stopped* on Day k if Water does not fill any new cells after Day k. Prove that if the flood has stopped and Water occupies m^2 cells, then there are at least $2m$ Rock cells in the landscape.

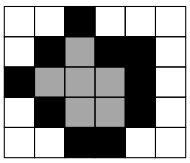

FIGURE 3. The flood has stopped.

(4) If the flood has stopped and Rock occupies at most N cells, determine the largest possible number of Water cells. Prove your answer.
(5) On Day 1, there are no Rock cells, and there are four Water cells forming a 2x2 square. Every day, you are allowed to replace 3 Ground cells with Rock cells. Note that you cannot put Rocks in Water.

Show how you can stop the flood.

		R3	R3	R4	
	R3	W3	W3	W4	R4
	R2	W2	W2	W3	R3
R2	W2	W1	W1	R1	
R2	W2	W1	W1	R1	
	R2	R1	R1		

FIGURE 4. Task 5: stopping the flood in four days, if 4 Rock cells per day are allowed. We write $W3$ if this cell fills with water on Day 3, and $R2$ if we put a Rock there on Day 2.

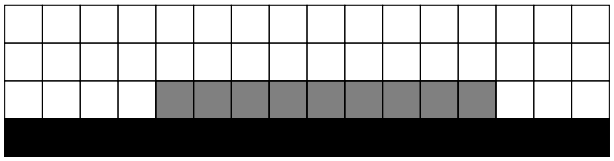

FIGURE 5. Day 1 configuration in Task 6 for $n = 9$

(6) On Day 1, there is an infinite horizontal row of Rock cells. Right above them, there is a row of adjacent n Water cells. Every day, you are allowed to replace k Ground cells with Rock cells. For any n, stop the flood for (a) $k = 3$; (b) $k = 2$.
(7) On Day 1, there are no Rock cells, and finitely many Water cells. Every day, you are allowed to replace k Ground cells with Rock cells. For any initial configuration of Water cells, prove that you can stop the flood for (a) k=9; (b) k=5; (c*) k=4; (d*) k=3.