Solutions to AB Exam

Texas A&M High School Math Contest

25 October, 2025

1. A food truck sells tacos only in boxes of 3, 4 and 10 tacos. How many boxes must one order to buy exactly 155 tacos?

   Answer: 18 (accepted any number larger than 18).

   Solution: We will find the minimum number of boxes needed. First try to use as many boxes of 10 as possible. If we buy 15 boxes of 10 tacos, then we're at 150, but there is no way to use multiples of 3 and/or 4 to buy the last 5. If we instead buy 14 boxes of 10 tacos, then we're at 140, leaving 15 more tacos to buy. In an attempt to use as many boxes of 4 as possible, we buy 3 boxes of 4 tacos, for a total of 12 more tacos. The last 3 tacos can be bought with a single box of 3. The minimum total number of boxes is $14 + 3 + 1 = 18$.

2. A bookshelf with 4 shelves holds 100 books. There are 5 more books on the second shelf than the first shelf. There are 2 more books on the third shelf than the second shelf. There are 7 fewer books on the fourth shelf than the third shelf. How many books are on the third shelf?

   Answer: 29 books

   Solution: Let x be the number of books on the first shelf. Then

$$\begin{aligned} x + (x+5) + (x+5+2) + (x+5+2-7) &= 100 \\ 4x + 12 &= 100 \\ 4x &= 88 \\ x &= 22 \end{aligned}$$

   The third shelf has $x + 7 = 29$ books.

3. Given that all numbers in the equation below are in base 6, find the value of x (in base 6) which solves the equation:

$$\frac{5(x-24)}{4} = 14$$

   Answer: 40

   Solution:

$$x = \left(14 \cdot \frac{4}{5} + 24\right)_6$$

   $14_6 = 10$, so

$$\left(14 \cdot \frac{4}{5}\right)_6 = \left(\frac{14}{5} \cdot 4\right)_6 = (2 \cdot 4)_6 = 12_6$$

$$(12 + 24)_6 = 40_6$$

4. A store is closing down and selling all items at half price. Paying with cash gives an additional 4% discount on sale prices. If you pay in cash, what is the overall percent discount from the original price?

   Answer: 52%

   Solution: Let P be the original purchase price. With the half-price sale, the price becomes $0.5P$. With the 4% cash discount, we are paying $0.96 \cdot 0.5P = 0.48P$. Thus we are paying 48% of the original price, meaning 52% off.

5. Five years ago, David was 4 times as old as Andrew and James was 6 times as old as Andrew. Today, David and Andrew's combined age is 4 years older than James. What will David, Andrew, and James's combined ages be in 2 years?

   Answer: 32 years

   Solution: Let D, A, J be the current ages of David, Andrew, and James respectively. Our system of equations is

$$\begin{aligned} D - 5 &= 4(A - 5) \\ J - 5 &= 6(A - 5) \\ D + A &= J + 4 \end{aligned}$$

   Solving the first two equations for D and J and substituting into the third gives us $(4A - 15) + A = 6A - 21$, which means $A = 6$. This means $D = 9$ and $J = 11$, so in two years, David, Andrew, and James will be 11, 8, and 13 respectively, for a sum of 32 years.

6. The points $A(1, 1)$, $B(5, 1)$, and $C(8, y)$, where $y > 1$, form a triangle with area 21. Find y.

   Answer: $\dfrac{23}{2}$.

   Solution: The triangle has area

$$\frac{1}{2}bh = \frac{1}{2}(5 - 1)(y - 1) = 2(y - 1) = 2y - 2$$

   We are given that the area is 21, thus

$$\begin{aligned} 2y - 2 &= 21 \\ 2y &= 23 \\ y &= \frac{23}{2} \end{aligned}$$

7. A drone flies along a straight path for a total of 100 miles in 4 hours. For the first third of the time, it maintains a constant speed of 10 miles per hour. For the remaining time, it flies at a different constant speed. What is this second speed? Give your final answer as a decimal.

   Answer: 32.5 mph

Solution: During the first third of the trip, the drone has flown for $\frac{4}{3}$ hours a total distance of $\frac{4}{3} \cdot 10 = \frac{40}{3}$ miles. For the last two thirds of the trip the drone flies for $\frac{8}{3}$ hours over a distance of $100 - \frac{40}{3} = \frac{260}{3}$ miles. Thus the speed was

$$\frac{\frac{260}{3}}{\frac{8}{3}} = \frac{260}{3} \cdot \frac{3}{8} = \frac{260}{8} = \frac{130}{4} = \frac{65}{2} = 32.5\text{mph}$$

8. When the number $14^5 \cdot 25^{20} \cdot 40^{11}$ is written out, how many consecutive zeroes does the number end with?

   Answer: 38

   Solution: We need to find how many times 10 divides this number:

$$14^5 \cdot 25^{20} \cdot 40^{11} = (2 \cdot 7)^5 \cdot (5^2)^{20} \cdot (2^3 \cdot 5)^{11} = 2^5 \cdot 7^5 \cdot 5^{40} \cdot 2^{33} \cdot 5^{11} = 2^{38} \cdot 5^{51} \cdot 7^5$$

   Since $10 = 2 \cdot 5$, we factor out as many 2's and 5's as possible, which is the minimum of their powers 38 and 51. Thus 10 divides this number 38 times, giving 38 consecutive zeroes at the end.

9. Five friends are playing a board game. After three rounds, their scores have an average of 12 points. One friend notices that if Alex's score were doubled, the group average would increase to 14 points. What was Alex's score?

   Answer: 10.

   Solution: Let their scores be A, B, C, D, E, with Alex represented by A. The average is 12, so

$$\frac{A+B+C+D+E}{5} = 12 \implies A+B+C+D+E = 60$$

   If Alex's score is doubled, the total is

$$2A+B+C+D+E = A + (A+B+C+D+E) = A + 60$$

   The new average is 14, so

$$\begin{aligned} \frac{A+60}{5} &= 14 \\ A + 60 &= 70 \\ A &= 10 \end{aligned}$$

10. Find the largest value of n so that 27^n divides $(2025)^{25}$.

    Answer: $33\frac{1}{3}$

    Solution: $2025 = (45)^2 = 3^4 \cdot 5^2$, so $(2025)^{25} = 3^{100} \cdot 5^{50}$. $27^n = 3^{3n}$, so to divide $(2025)^{25}$, we need $3n \leq 100$. The largest such value of n is 33.

11. Let k be a positive constant. Given the system of equations below, find the value of k which makes $kx + y = 0$.

$$kx + 4y = 12$$
$$x + \frac{2}{k}y = 6$$

Answer: $\frac{2}{3}$.

Solution: We need $3y = 12$, so $y = 4$. Our system of equations becomes

$$kx + 16 = 12$$
$$x + \frac{8}{k} = 6$$

Multiplying the second equation by k and subtracting the first equation gives us $-8 = 6k - 12$, so $k = \frac{2}{3}$. This means $(x, y) = (-6, 4)$ is a solution to the system of equations, and $kx + y = \frac{2}{3}(-6) + 4 = 0$.

12. A group of robots work in a factory. A robot's light is green when it is charging and blue when it is working. Initially, the ratio of green to blue robots was $3 : 1$. Then 6 green robots stopped charging and started working, and 4 blue robots started charging. After these changes, the ratio of green to blue robots became $5 : 2$. What is the difference between the number of green robots and the number of blue robots now?

Answer: 24

Solution: Let k be the initial number of blue robots, then the ratio of $3 : 1$ means there are initially $3k$ green robots. After the changes, there are $3k - 6 + 4 = 3k - 2$ green robots and $k + 6 - 4 = k + 2$ blue robots. The new ratio of $5 : 2$ gives

$$\begin{aligned} \frac{3k-2}{k+2} &= \frac{5}{2} \\ 2(3k-2) &= 5(k+2) \\ 6k - 4 &= 5k + 10 \\ k &= 14 \end{aligned}$$

There are now $3k - 2 = 3(14) - 2 = 40$ green robots and $k + 2 = 14 + 2 = 16$ blue robots. The difference is $40 - 16 = 24$.

13. Solve for x: $3 - \sqrt{3x + 10} = x$. If necessary, write your simplified answer(s) in the form of $a + b\sqrt{c}$

Answer: $\frac{9}{2} - \frac{1}{2}\sqrt{85}$

Solution: Isolating the radical gives us $3 - x = \sqrt{3x + 10}$; hence, $3 - x \geq 0$, or $x \leq 3$.

Square both sides: $x^2 - 6x + 9 = 3x + 10$, or $x^2 - 9x - 1 = 0$.

From the quadratic formula, $x = \dfrac{9 \pm \sqrt{85}}{2}$. However, $\dfrac{9 + \sqrt{85}}{2} > 3$, which contradicts our initial restriction on x. So our solution, in the required form, is $x = \dfrac{9}{2} - \dfrac{1}{2}\sqrt{85}$.

14. How many three-digit numbers are divisible by 11?

    Answer: 81

    Solution: Such a number is of the form $11k$ for some integer k. To be a three-digit number, we need $100 \le 11k \le 999$. Solving $100 \le 11k$ gives $\frac{100}{11} \le k$, so $10 \le k$. Solving $11k \le 999$ gives $k \le 90$. Thus $10 \le k \le 90$ and there are $90 - 10 + 1 = 81$ such three-digit numbers.

15. Working at a constant rate, Bob the Builder would need 30 hours to build a certain brick wall. Working at a different constant rate, Barb the Builder would need 21 hours to build the same wall. Working together, though, they can each lay an extra 4 bricks per hour and build the wall in 11 hours and 40 minutes. How many bricks are in the wall?

    Answer: 1680 bricks.

    Solution: Let x be the number of bricks in the wall. Working by themselves, Bob lays down $\dfrac{x}{30}$ bricks per hour and Barb lays down $\dfrac{x}{21}$ bricks per hour. Working together, they lay down $\left(\dfrac{x}{30} + 4\right) + \left(\dfrac{x}{21} + 4\right) = \dfrac{x}{30} + \dfrac{x}{21} + 8$ bricks per hour, so the amount of time it would take them to build the wall is

    $$\frac{x}{\frac{x}{30} + \frac{x}{21} + 8} = 11\frac{2}{3} = \frac{35}{3}$$

    Cross-multiply to obtain

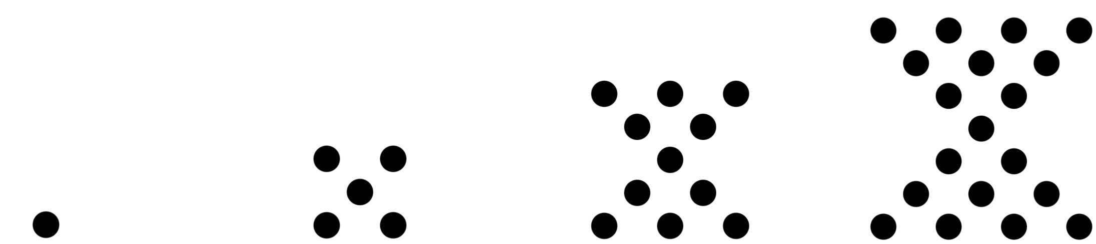
    $$3x = \frac{7}{6}x + \frac{5}{3}x + 280$$

    Multiply by 6 to clear the fractions

    $$18x = 7x + 10x + 1680$$

    So $x = 1680$ bricks.

16. Define "hourglass numbers" H_n according to the images below.



$H_1 = 1$ $H_2 = 5$ $H_3 = 11$ $H_4 = 19$

If the pattern continues, find H_{25}.

Answer: 649

Solution: The 25th hourglass number can be written as

$$25 + 24 + 23 + \cdots + 2 + 1 + 2 + \cdots + 23 + 24 + 25$$

$$= (25 + 1) + (24 + 2) + (23 + 3) + \cdots + (2 + 24) + (1 + 25) - 1$$

There are 25 pairings listed, so $H_{25} = 25(26) - 1 = 650 - 1 = 649$.

17. The **geometric mean** of two positive numbers a and b is a positive number x such that $x^2 = ab$. If the average of two numbers a and b (with $a > b > 0$) is twice as large as their geometric mean, what is $\frac{a}{b}$ rounded to the nearest whole number?

Answer: 14

Solution: The average of the numbers is $\frac{a+b}{2} = 2\sqrt{ab}$, so

$$a + b = 4\sqrt{ab}$$

Square both sides to obtain

$$a^2 + 2ab + b^2 = 16ab$$

$$\left(\frac{a}{b}\right)^2 - 14\left(\frac{a}{b}\right) + 1 = 0$$

Solve for $\frac{a}{b}$ using the quadratic formula:

$$\frac{a}{b} = \frac{14 \pm \sqrt{(14)^2 - 4(1)}}{2}$$

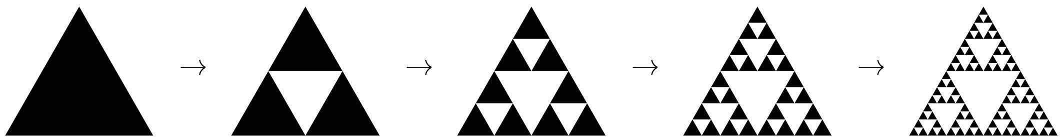
$$\frac{a}{b} = \frac{14 \pm \sqrt{192}}{2}$$

$\frac{a}{b} > 1$, so $\frac{a}{b} = \frac{14 + \sqrt{192}}{2}$. Since $13 < \sqrt{192} < 14$, we have

$$\frac{27}{2} < \frac{a}{b} < 14$$

So to the nearest integer $\frac{a}{b} = 14$.

18. Consider a filled in equilateral triangle with area $\sqrt{3}$. Connect the midpoints of each side to form a new, inverted triangle in the center, and remove this area from the original triangle. Repeat this process as in the image below. Find the filled-in area of the last image (after applying this process 4 times).



Answer: $\frac{81}{256}\sqrt{3}$

Solution: Let A_n represent the filled-in area after the nth iteration and $A_0 = \sqrt{3}$. After each iteration, the next filled-in area is $\frac{3}{4}$ of the previous, which means $A_n = \left(\frac{3}{4}\right)^n A_0 = \left(\frac{3}{4}\right)^n \sqrt{3}$. Thus

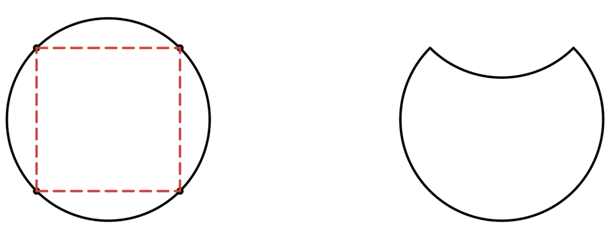
$$A_4 = \left(\frac{3}{4}\right)^4 \sqrt{3} = \frac{81}{256}\sqrt{3}$$

19. A line segment is drawn in the x-y plane from the point (500, 1000) to the point (1000, 2025). How many points on the line segment have integer coordinates?

Answer: 26

Solution: We can translate the origin of our coordinate system to the first point, meaning our points have new coordinates of (0, 0) and (500, 1025). The line segment has a slope of $\frac{1025}{500}$, which reduces (by a factor of 25) to $\frac{41}{20}$. Since these numbers have no common factor larger than 1, the additional points with integer coordinates on our line segment must be (20, 41), (40, 82), (60, 123), $\cdots$ (480, 984), giving us 26 points in all.,

20. We start with a circle of radius 2 and inscribe a square inside it. We then reflect the arc of the circle above the square about the top edge of the square to obtain the following shape. Find its area.



Answer: $2\pi + 4$

Solution: The area of the circle is $\pi(2)^2 = 4\pi$. A diagonal of the square corresponds to a diameter measurement, so has length 4. The Pythagorean Theorem gives the side length of each square as $\sqrt{8}$. The area of the square is then $(\sqrt{8})^2 = 8$. The four pieces of the circle lying outside the square each have area $\frac{4\pi-8}{4} = \pi - 2$. The shape whose area we are calculating amounts to subtracting two of these pieces from the original circle, giving an area of $4\pi - 2(\pi - 2) = 2\pi + 4$.

21. Solve for x: $2^{2^x} = 4^{4^x}$

Answer: -1

Solution: Note that $4^{4^x} = (2^2)^{(2^2)^x} = (2^2)^{2^{2x}} = 2^{2 \cdot 2^{2x}} = 2^{2^{2x+1}}$.

Therefore,

$$2^{2^x} = 2^{2^{2x+1}}$$

$$2^x = 2^{2x+1}$$

$$x = 2x + 1 \Rightarrow x = -1$$

22. Find the largest solution x of the equation $\big|3|x| - 2\big| = 1 - 2x$.

Answer: $-1/5$

Solution: First consider the case $x \geq 0$. In this case $|x| = x$ and the equation is simplified to $|3x - 2| = 1 - 2x$. Since $3x - 2 = 0$ for $x = 2/3$, we need to consider two subcases: $0 \leq x < 2/3$ and $x \geq 2/3$.

If $x \geq 2/3$ then the equation is further simplified to $3x - 2 = 1 - 2x$, which has solution $x = 3/5$. However, we have to drop this solution (at least in this subcase) since it does not belong to the interval $[2/3, \infty)$.

If $0 \leq x < 2/3$ then the equation becomes $-(3x - 2) = 1 - 2x$, which has solution $x = 1$. This one has to be dropped too as it does not belong to the interval $[0, 2/3)$.

Now consider the case $x < 0$. In this case $|x| = -x$ and the equation is simplified to $|-3x - 2| = 1 - 2x$, which is equivalent to $|3x + 2| = 1 - 2x$. Again, there are two subcases to consider: $-2/3 \leq x < 0$ and $x < -2/3$.

If $-2/3 \leq x < 0$ then the equation is further simplified to $3x + 2 = 1 - 2x$, which has solution $x = -1/5$. This time $-1/5$ does belong to the interval of interest $[-2/3, 0)$. Hence it is also a solution of the original equation.

Finally, if $x < -2/3$, any solution will be smaller than $-1/5$. Therefore, the largest solution is $-1/5$.