Solutions to BC Exam

Texas A&M High School Math Contest

25 October, 2025

1. Find the x-intercepts for $y = 36x^2 + 81xa - 81a^2$ if constant $a > 0$.

   Answer: $(-3a,\, 0)$ and $\left(\dfrac{3}{4}a,\, 0\right)$.

   Solution: $0 = 36x^2 + 81xa - 81a^2$. The quadratic factors into $9(x+3a)(4x-3a)$, so our x-intercepts are $(-3a,\, 0)$ and $\left(\dfrac{3}{4}a,\, 0\right)$

2. Reveille decides to enter a doggy triathlon. Reveille dog-paddles 1 mile in 15 minutes, "revs" the bicycle at a rate of 10 mph for 30 minutes, and runs 4 miles at a rate of 20 mph. What is the average speed (in mph) for Reveille during the event?

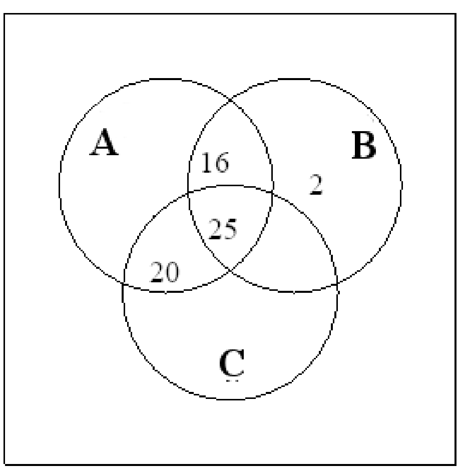
   Answer: $\dfrac{200}{19}$ mph

   Solution: Reveille runs 1 mile in 15 minutes, bicycles 5 miles in 30 minutes and runs 4 miles in 12 minutes (1/5 of an hour). So Reveille traveled a total of 10 miles in 57 minutes $= \dfrac{19}{20}$ of an hour, so her average speed was $\dfrac{200}{19}$ mph

3. Teens are divided about the best commonly used AI platform. 83 percent like ChatGPT, 52 percent like Copilot, and 57 percent like Gemini. However, 25 percent like all 3 platforms, 16 percent like ChatGPT and Copilot only, and 20 percent like ChatGPT and Gemini only. If 2 percent exclusively use Copilot, what percentage of teens do not like any of the 3 AI platforms?

   Answer: 3%

   Solution: Let A be the set of students who like ChatGPT, B be the set of students who like Copilot, and C be the set of students who like Gemini. The Venn Diagram with inital data is shown below.



The percentage exclusively in set A is $83-20-25-16=22$, the percentage exclusively in sets B and C is $52-16-25-2=9$, and the percentage exclusively in set C is $57-20-25-9=3$. Adding all of these percentages together gives us 97%, so 3% of teens do not like any of the 3 AI platforms.

4. Given that all numbers in the equation below are in base 6, find the value of x (in base 6) which solves the equation:

$$\frac{5(x-24)}{4}=14$$

Answer: 40

Solution:

$$x=\left(14\cdot\frac{4}{5}+24\right)_6$$

$14_6=10$, so

$$\left(14\cdot\frac{4}{5}\right)_6=\left(\frac{14}{5}\cdot 4\right)_6=(2\cdot 4)_6=12_6$$

$$(12+24)_6=40_6$$

5. In a triangle ABC with $AB=5, BC=6$, and $AC=7$, points D and E lie on $\overline{AC}$ with $AD=1$ and $EC=2$. Find the area of $\triangle BDE$.

Answer: $\dfrac{24\sqrt{6}}{7}$

Solution: Let s be the semiperimeter and $[\triangle ABC]$ be the area of $\triangle ABC$.

$$s=\frac{AB+BC+CA}{2}=\frac{5+6+7}{2}=9.$$

By Heron's formula,

$$[\triangle ABC]=\sqrt{9(9-5)(9-6)(9-7)}=\sqrt{216}=6\sqrt{6}.$$

Thus,

$$\frac{[\triangle BDE]}{[\triangle BAC]}=\frac{DE}{AC}=\frac{4}{7},\qquad\text{or}\qquad[\triangle BDE]=6\sqrt{6}\cdot\frac{4}{7}=\frac{24\sqrt{6}}{7}.$$

6. Five years ago, David was 4 times as old as Andrew and James was 6 times as old as Andrew. Today, David and Andrew's combined age is 4 years older than James. What will David, Andrew, and James's combined ages be in 2 years?

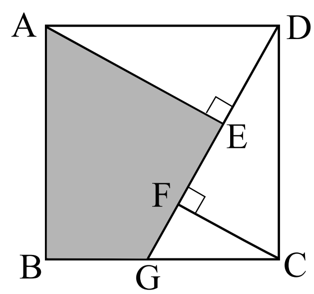
Answer: 32 years

Solution: Let D, A, J be the current ages of David, Andrew, and James respectively. Our system of equations is

$$D - 5 = 4(A - 5)$$
$$J - 5 = 6(A - 5)$$
$$D + A = J + 4$$

Solving the first two equations for D and J and substituting into the third gives us $(4A - 15) + A = 6A - 21$, which means $A = 6$. This means $D = 9$ and $J = 11$, so in two years, David, Andrew, and James will be 11, 8, and 13 respectively, for a sum of 32 years.

7. The figure below shows a square $ABCD$. Find the area of the shaded region if $EF = 3$ and $DG = 13$.



Answer: 51.

Solution: Since $\triangle ADE$ is congruent to $\triangle DCF$, $DE = FC$ and $AE = DF$. Let $DE = x$ then $FG = 10 - x$. From the similarity $\triangle DFC \sim \triangle CFG$, we have

$$\frac{FC}{DF} = \frac{FG}{FC} \quad \Rightarrow \quad \frac{x}{3+x} = \frac{10-x}{x} \quad \Rightarrow \quad 2x^2 - 7x - 30 = (x-6)(2x+5) = 0 \quad \Rightarrow \quad x = 6$$

We have $AE = DF = 6 + 3 = 9$ and

$$DC = AD = \sqrt{6^2 + 9^2} = 3\sqrt{13}, \quad GC = \frac{2}{3}DC = 2\sqrt{13}$$

The area of shaded region is

$$(3\sqrt{13})^2 - \frac{AE \cdot ED}{2} - \frac{GC \cdot CD}{2} = 117 - 27 - \frac{2\sqrt{13} \cdot 3\sqrt{13}}{2} = 51.$$

8. Among the 9 points with integer coordinates (x, y) with $0 \le x \le 2$ and $0 \le y \le 2$, how many distinct pentagons can be formed by choosing 5 of these points as vertices such that all interior angles are strictly less than $180°$?

Answer: 20

Solution: Label the 3×3 grid points. The center point $(1, 1)$ cannot be a vertex of a pentagon with the condition. Hence the vertices must be chosen from the 8 outer points.

A 5-point subset of these 8 outer points forms a convex pentagon if and only if no three of the chosen points are collinear. Among the outer 8 points, the only collinear triples are the four side-triples of the bounding square:

$$\{(0,0),(1,0),(2,0)\}, \quad \{(0,2),(1,2),(2,2)\}, \quad \{(0,0),(0,1),(0,2)\}, \quad \{(2,0),(2,1),(2,2)\}.$$

There are $\binom{8}{5} = 56$ ways to choose 5 of the outer 8 points in total. Let $A_1, \ldots, A_4$ be the events that our 5-set contains all three points of the bottom, top, left, and right side, respectively. Then $|A_i| = \binom{5}{2} = 10$ for each i (after fixing one side's triple, choose the remaining 2 from the other 5 points), so $\sum |A_i| = 4 \cdot 10 = 40$.

We consider intersections of events. Two opposite sides cannot both be fully included in a 5-set, but two *adjacent* sides can; in that case the 5 points are forced (the union of the two side-triples has size 5), so $|A_i \cap A_j| = 1$ for each of the 4 adjacent pairs, giving $\sum_{i<j} |A_i \cap A_j| = 4$. Any intersection of 3 or 4 sides is impossible in a 5-set.

By inclusion–exclusion, the number of valid 5-subsets is

$$\binom{8}{5} - \sum |A_i| + \sum_{i<j} |A_i \cap A_j| = 56 - 40 + 4 = 20.$$

9. If $3kx + 4y = 7$ intersects $(6+k)x + y = 2$ when $y = 19$, find k.

Answer: -23

Solution: Substituting $y = 19$ into our equations gives us

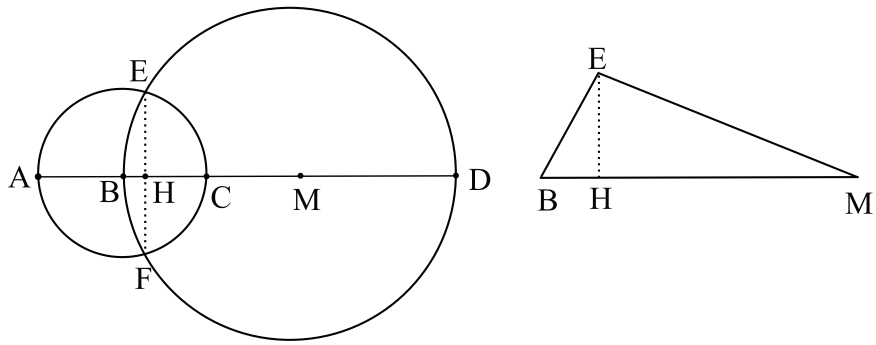
$$\begin{aligned} 3kx &= -69 \\ (6+k)x &= -17 \end{aligned}$$

Mulitplying the first equation by 17 and the second equation by 69 gives us the same constant on the right, so $51kx = (414 + 69k)x$. $x = 0$ is clearly not a solution to the original equations, so we have $51k = 414 + 69k$, which is true when $k = -23$.

10. Four points A, B, C, D lie on a straight line in that order, such that $AB = BC = 1$ and $CD = 3$. Let E and F be the two intersection points of the circle with diameter $\overline{AC}$ and the circle with diameter $\overline{BD}$. Find the area of quadrilateral $EAFD$.

Answer: $\dfrac{5\sqrt{15}}{4}$.

Solution: Set C_1 to be the circle with diameter $\overline{AC}$ and C_2 the circle with diameter $\overline{BD}$. Let M be the midpoint of $\overline{BD}$ and H be the intersection of $\overline{EF}$ and $\overline{AB}$. Two circles have radii $BE = BC = 1$ and $ME = BM = 2$ respectively.



With $BH = x$, apply the Pythagorean theorem to two right triangles sharing $\overline{EH}$ to have

$$1 - x^2 = 2^2 - (2 - x)^2 \quad \Rightarrow \quad x = \frac{1}{4},$$

which implies

$$EH = \sqrt{1 - \frac{1}{16}} = \frac{\sqrt{15}}{4}$$

The area of quadrilateral $EAFD$ becomes

$$S = 2\,\frac{1}{2}\,AD \cdot EH = \frac{5\sqrt{15}}{4}$$

11. Find the largest value of n so that 27^n divides $(2025)^{25}$.

    Answer: $33\frac{1}{3}$

    Solution: $2025 = (45)^2 = 3^4 \cdot 5^2$, so $(2025)^{25} = 3^{100} \cdot 5^{50}$. $27^n = 3^{3n}$, so to divide $(2025)^{25}$, we need $3n \leq 100$. The largest such value of n is $33\frac{1}{3}$.

12. Let k be a positive constant. Given the system of equations below, find the value of k which makes $kx + y = 0$.

$$kx + 4y = 12$$
$$x + \frac{2}{k}y = 6$$

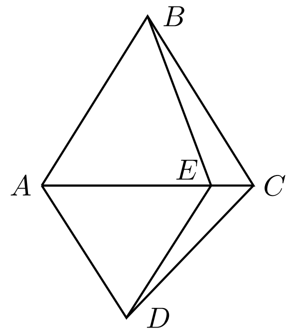
Answer: $\dfrac{2}{3}$.

Solution: We need $3y = 12$, so $y = 4$. Our system of equations becomes

$$kx + 16 = 12$$
$$x + \frac{8}{k} = 6$$

Multiplying the second equation by k and subtracting the first equation gives us $-8 = 6k - 12$, so $k = \dfrac{2}{3}$. This means $(x, y) = (-6, 4)$ is a solution to the system of equations, and $kx + y = \dfrac{2}{3}(-6) + 4 = 0$.

13. For two equilateral triangles $\triangle ABC$ and $\triangle ADE$, find $\angle BEC$ if $\angle CDE = 14°$.



Answer: 106°

Solution: Since $AB = AC$, $AE = AD$, and $\angle BAE \cong \angle CAD$, we have congruent triangles $\triangle BAE \cong \triangle CAD$. ($\triangle CAD$ is the rotation of $\triangle BAE$ by 60° clockwise) This implies $\angle AEB \cong \angle ADC = 60° + 14° = 74°$, or $\angle BEC = 180° - 74° = 106°$.

14. Solve for x: $3 - \sqrt{3x + 10} = x$. If necessary, write your simplified answer(s) in the form of $a + b\sqrt{c}$

Answer: $\frac{9}{2} - \frac{1}{2}\sqrt{85}$

Solution: Isolating the radical gives us $3 - x = \sqrt{3x + 10}$; hence, $3 - x \geq 0$, or $x \leq 3$.
Square both sides: $x^2 - 6x + 9 = 3x + 10$, or $x^2 - 9x - 1 = 0$.
From the quadratic formula, $x = \dfrac{9 \pm \sqrt{85}}{2}$. However, $\dfrac{9 + \sqrt{85}}{2} > 3$, which contradicts our initial restriction on x. So our solution, in the required form, is $x = \frac{9}{2} - \frac{1}{2}\sqrt{85}$.

15. The **geometric mean** of two positive numbers a and b is a positive number x such that $x^2 = ab$. If the average of two numbers a and b (with $a > b > 0$) is twice as large as their geometric mean, what is $\frac{a}{b}$ rounded to the nearest whole number?

Answer: 14

Solution: The average of the numbers is $\dfrac{a+b}{2} = 2\sqrt{ab}$, so

$$a + b = 4\sqrt{ab}$$

Square both sides to obtain

$$a^2 + 2ab + b^2 = 16ab$$
$$\left(\frac{a}{b}\right)^2 - 14\left(\frac{a}{b}\right) + 1 = 0$$

Solve for $\frac{a}{b}$ using the quadratic formula:

$$\frac{a}{b} = \frac{14 \pm \sqrt{(14)^2 - 4(1)}}{2}$$

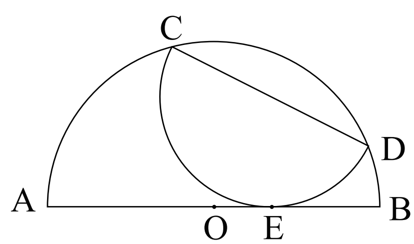
$$\frac{a}{b} = \frac{14 \pm \sqrt{192}}{2}$$

$\frac{a}{b} > 1$, so $\frac{a}{b} = \frac{14 + \sqrt{192}}{2}$. Since $13 < \sqrt{192} < 14$, we have

$$\frac{27}{2} < \frac{a}{b} < 14$$

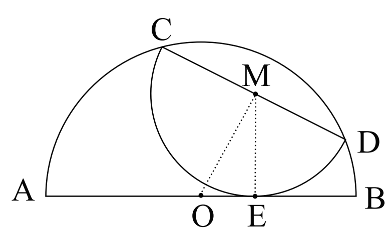
So to the nearest integer $\frac{a}{b} = 14$.

16. Points C and D lie on the arc of a semicircle with diameter $\overline{AB}$. Let O be the midpoint of $\overline{AB}$. The circle with diameter $\overline{CD}$ is tangent to the segment $\overline{AB}$ at a point E as in the figure below. Given $CD = 12$ and $OE = 1$, find the value of AB^2.



Answer: 292

Solution: Let the semicircle S_1 have center $O = (0, 0)$ and radius R. Without loss of generality, we may assume the the configuration of two semi circles as in the figure below since the tangency point E has coordinate either $E = (-1, 0)$ or $E = (0, 1)$ and it doesn't affect the AB.



Let M be the midpoint of $\overline{CD}$. The semicircle S_2 with diameter $\overline{CD}$ has the center M and radius $r = \frac{CD}{2} = 6$. Tangency to the line $\overline{AB}$ (the x-axis) implies that the perpendicular distance from M to $\overline{AB}$ equals r; hence $ME = 6$.

On $\triangle MOE$, we have

$$OM^2 = 1^2 + 6^2 = 37.$$

Moreover, on the cord $\overline{CD}$ of S_1 (or on the right triangle $\triangle OMC$ or $\triangle OMD$), we have

$$R^2 = OM^2 + OC^2 = 37 + OA^2 = 37 + 6^2 = 73.$$

Therefore, $AB = 2R$ and

$$AB^2 = (2R)^2 = 4R^2 = 4 \cdot 73 = 292.$$

17. A line segment is drawn in the x-y plane from the point (500, 1000) to the point (1000, 2025). How many points on the line segment have integer coordinates?

    Answer: 26

    Solution: We can translate the origin of our coordinate system to the first point, meaning our points have new coordinates of (0, 0) and (500, 1025). The line segment has a slope of $\dfrac{1025}{500}$, which reduces (by a factor of 25) to $\dfrac{41}{20}$. Since these numbers have no common factor larger than 1, the additional points with integer coordinates on our line segment must be (20, 41), (40, 82), (60, 123), $\cdots$ (480, 984), giving us 26 points in all.,

18. Find the largest solution x of the equation $\big|3|x| - 2\big| = 1 - 2x$.

    Answer: $-1/5$

    Solution: First consider the case $x \geq 0$. In this case $|x| = x$ and the equation is simplified to $|3x - 2| = 1 - 2x$. Since $3x - 2 = 0$ for $x = 2/3$, we need to consider two subcases: $0 \leq x < 2/3$ and $x \geq 2/3$.

    If $x \geq 2/3$ then the equation is further simplified to $3x - 2 = 1 - 2x$, which has solution $x = 3/5$. However, we have to drop this solution (at least in this subcase) since it does not belong to the interval $[2/3, \infty)$.

    If $0 \leq x < 2/3$ then the equation becomes $-(3x - 2) = 1 - 2x$, which has solution $x = 1$. This one has to be dropped too as it does not belong to the interval $[0, 2/3)$.

    Now consider the case $x < 0$. In this case $|x| = -x$ and the equation is simplified to $|-3x - 2| = 1 - 2x$, which is equivalent to $|3x + 2| = 1 - 2x$. Again, there are two subcases to consider: $-2/3 \leq x < 0$ and $x < -2/3$.

    If $-2/3 \leq x < 0$ then the equation is further simplified to $3x + 2 = 1 - 2x$, which has solution $x = -1/5$. This time $-1/5$ does belong to the interval of interest $[-2/3, 0)$. Hence it is also a solution of the original equation.

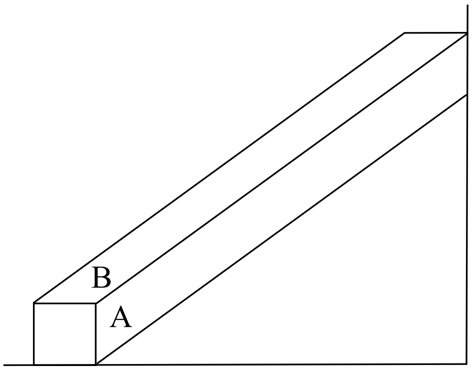
    Finally, if $x < -2/3$, any solution will be smaller than $-1/5$. Therefore, the largest solution is $-1/5$.

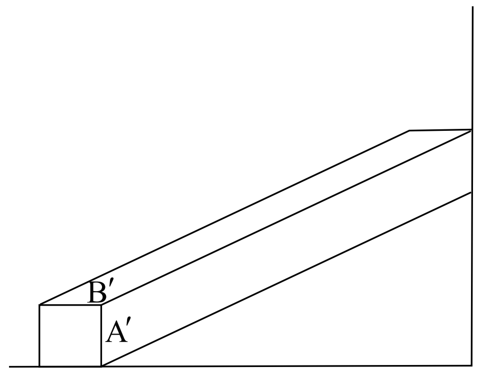
19. A cube moves along a straight line perpendicular to the front face, so that it collects rain only on its top and front faces—not on the lateral faces. The cube travels 12 meters with an initial speed of 1 m/s, while rain falls vertically at 6 m/s and is evenly distributed. If the cube's speed is increased by 50% to reduce the amount of rain collected, what is the percentage decrease in the total rain collected? Round your answer to the nearest tenth of a percent.

Answer: 28.6%

Solution: The amount of rain collected is determined by the volume of space swept by the cube's top and front faces. Since raindrops are evenly distributed, comparing these volumes is sufficient.

Consider the rain that hits the front face during the journey. The raindrops are contained within the space modeled as parallelogram A shown in the figure on the left.





This parallelogram A has an area (representing volume) of $s \cdot 12$, where s is the side of the cube. Similarly, the rain that hit the top face are in the space corresponding to the parallelogram B. The area of B is

$$s \cdot 12 \cdot 6 = 72s.$$

The right figure illustrates parallelograms A' and B' that contain rain hitting the front and top faces, respectively, with the increased speed of 1.5 m/s.

The area of A' is the same as the area of A while B' has the area

$$s \cdot 8 \cdot 6 = 48s.$$

Now we compare the amount of rain the cube collects at the new speed $R_{1.5}$ to the initial amount R_1:

$$R_{1.5} : R_1 = (12s + 48s) : (12s + 72s) = \frac{5}{7}$$

The increased speed reduces the amount of rain collected by 28.6%, since

$$\frac{2}{7} = 0.2857\cdots$$

20. From a point A outside a circle C, two tangents are drawn to the circle, touching it at points P and Q. Through P, draw a line parallel to $\overline{AQ}$, which meets the circle again at R. Let the line $\overline{AR}$ intersect the circle again at S. If $AP : PR = 2 : 3$ and the area of $\triangle ASQ$ is 50, find the area of quadrilateral $APRQ$.

Answer: 500

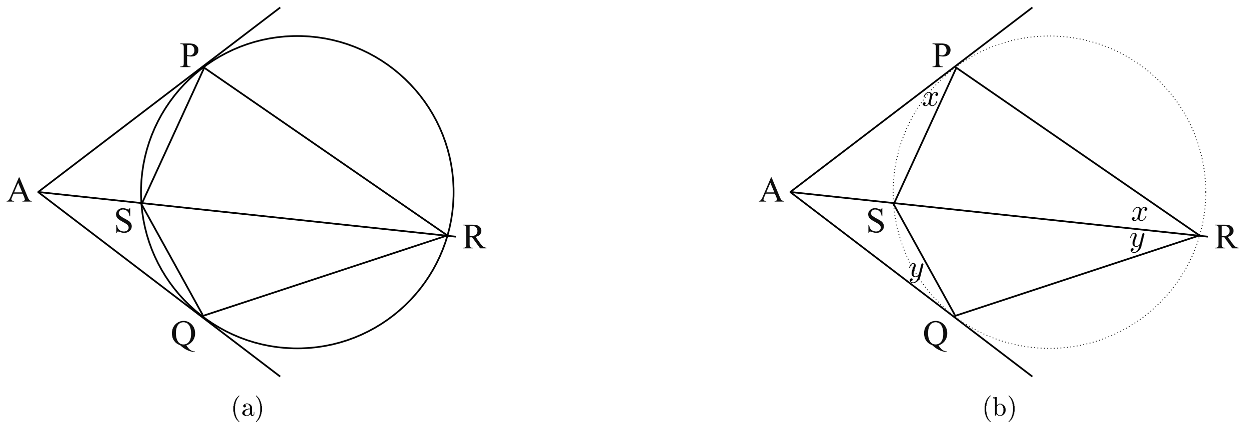


Figure 1: Initial stage with tangent and secant lines.

Solution: Figure 1 (a) illustrates the quadrilateral $APRQ$. As in Figure 1 (b), we draw secant lines $\overline{PS}$ and $\overline{SQ}$ to have

$$\angle PRA \cong \angle APS := x \text{ and } \angle QRA \cong \angle AQS := y$$

We need to examine angles further. A transversal $\overline{QR}$ passing parallel lines $\overline{PR}$ and $\overline{AQ}$ implies

$$\angle AQR = 180^\circ - \angle PRQ = 180^\circ - (x+y),$$

and so the sum of interior angles of $\triangle AQR$ yields

$$\angle QAR = 180^\circ - (\angle AQR + \angle QRA) = 180^\circ - (180^\circ - (x+y)) - y = x.$$

The sum of angels in the inscribed quadrilateral $\square PSQR$ is

$$\begin{aligned} &\angle RPS + \angle PSQ + \angle SQR + \angle QRP \\ =&\angle RPS + (180^\circ - (x+y)) + (180^\circ - (x+y) - y) + (x+y) = 360^\circ, \end{aligned}$$

which implies

$$\angle RPS = x + 2y. \tag{1}$$

Figure 2 (a) below shows angles in x and y that we have found so far.

As in Figure 2 (b) above, draw a secant line $\overline{PQ}$. By Tangent–Secant Angle Theorem applied to the tangents $\overline{AP}$ and $\overline{AQ}$, and a secant line $\overline{PQ}$, we have

$$\angle APQ \cong \angle AQP \cong \angle PRQ = x + y.$$

Consequently,

$$\angle SPQ = \angle APQ - \angle APS = (x+y) - x = y \tag{2}$$

and similarly,

$$\angle SQP = \angle AQP - \angle AQS = (x+y) - y = x$$

Now (2) and (3) imply

$$\angle RPQ = \angle RPS - \angle SPQ = x + y.$$

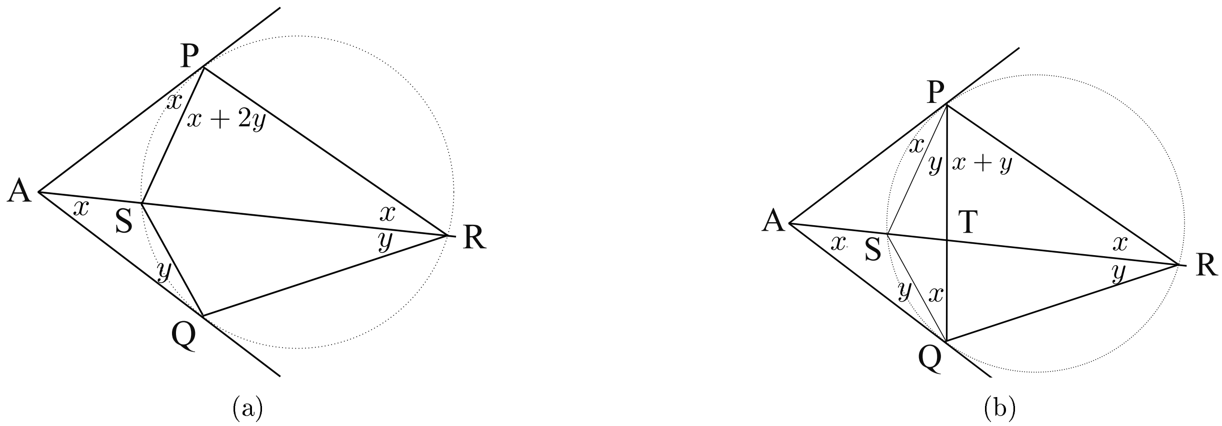


Figure 2: Further congruent angles

We have similar triangles

$$\triangle ASQ \sim \triangle QSP \text{ and } \triangle QPR \sim \triangle AQP \text{ (isosceles)} \tag{3}$$

together with

$$\triangle APR \sim \triangle ASP \tag{4}$$

The similarity (5) and the condition that $AP : PR = 2 : 3$ imply

$$\frac{AP}{PR} = \frac{2}{3} = \frac{AS}{PS}.$$

The first similarity of (4) implies

$$\frac{AS}{QS} = \frac{QS}{PS} \quad \Rightarrow \quad QS^2 = AS \cdot PS \quad \Rightarrow \quad \left(\frac{QS}{AS}\right)^2 = \frac{PS}{AS} = \frac{3}{2} \tag{5}$$

The area of $\triangle QSP$, $[\triangle QSP]$, satisfies

$$\frac{[\triangle QSP]}{[\triangle ASQ]} = \left(\frac{QS}{AS}\right)^2 \quad \Rightarrow \quad [\triangle QSP] = \frac{3}{2}50 = 75.$$

Transversal lines $\overline{AR}$ and $\overline{PQ}$ passing parallel lines $\overline{PR}$ and $\overline{AQ}$ determine a similar triangles $\triangle TPR \sim \triangle TQA$. Two tangent lines $\overline{AP}$ and $\overline{AQ}$ are congruent and so the similarity ratio implies

$$\frac{QT}{PT} = \frac{AQ}{PR} = \frac{2}{3} \quad \Rightarrow \quad \frac{[\triangle PST]}{[\triangle QST]} = \frac{AQ}{PR} = \frac{2}{3}.$$

Now we have the area of $\triangle AQT$:

$$[\triangle AQT] = [\triangle ASQ] + [\triangle QST] = 50 + \frac{2}{5}[\triangle QSP] = 50 + \frac{2}{5}75 = 80.$$

Similarly, the area of $\triangle PAT$ is

$$[\triangle AQT] = \frac{3}{2}[\triangle AQT] = 120.$$

Next, we find the area of $\triangle QPR$, which is determined by the second similarity ratio in (4). By the same argument as in (6), we have

$$\frac{AP}{PQ} = \frac{PQ}{PR} \quad \Rightarrow \quad \frac{PQ^2}{AP^2} = \frac{3}{2}$$

Therefore, the area of $\triangle QPR$ is

$$[\triangle QPR] = \frac{3}{2}[\triangle APQ] = \frac{3}{2}(80 + 120) = 300.$$

The area of quadrilateral $APRQ$ is

$$[\triangle APQ] + [\triangle QPR] = 200 + 300 = 500.$$