Best Student Exam
Texas A&M High School Math Contest
November, 2025

1. Find the minimal value of $\sqrt{x^2+(1-y)^2}+\sqrt{(1-x)^2+y^2}$, where x, y are real numbers.

2. In $\triangle ABC$ the median from A is perpendicular to the median from B. Find AB if $BC = 7$ and $AC = 6$.

3. Two particles move along the edges of equilateral $\triangle ABC$ in the direction $A \to B \to C \to A$ starting simultaneously and moving at the same speed. One starts at A, and the other starts at the midopoint of $\overline{BC}$. The midpoint of the line segment joining the two particles traces out a path that encloses a region R. What is the ratio of the area of R to the area of $\triangle ABC$?

4. A standard 8×8 chess board is rotated around its center by $45°$. Find the area of the set of the points of the plane which belonged to black squares in both the original and the rotation positions of the board, if the side of the board length $1ft$.

5. Let $\lfloor x \rfloor$ denote the largest integer not larger than x, and let $\{x\} = x - \lfloor x \rfloor$. Find the number of real solution of the equation

$$\lfloor x \rfloor \cdot \{x\} + x = 2\{x\} + 10.$$

6. Find all two-digit decimal numbers $\overline{xy}$ such that their square is equal to the cube of the sum of its digits (i.e., to $(x+y)^3$).

7. Let $\triangle ABC$ be such that the angle $\angle ACB$ is right, and the radius of the inscribed circle of $\triangle ABC$ is 1. Let CD be the height of $\triangle ABC$. Suppose that the bisector of $\angle ABC$ intersects the bisector of $\angle BCD$ in E, and that the bisector of $\angle BAC$ intersects the bisector of $\angle ACD$ in F. Find EF.

8. Three lines ℓ_1, ℓ_2, ℓ_3 are parallel to the sides of $\triangle ABC$ and intersect in one point in its interior. All three segments of ℓ_i formed by the two intersection points of ℓ_i with the sides of $\triangle ABC$ have the same length x. Find x if the lengths of the sides of $\triangle ABC$ are $2, 3, 4$.

9. For a positive integer n, let $S(n)$ denote the sum of decimal digits of n. Find the number of solutions of the equation $n + S(n) + S(S(n)) = 2025$.

10. How many numbers α there exist such that $0 \le \alpha < 2\pi$ and all numbers

$$\cos\alpha, \cos 2\alpha, \cos 4\alpha, \ldots, \cos 2^n\alpha, \ldots$$

are negative?

11. Evaluate $\int_0^{\pi}(|\sin 2025x| - |\sin 2026x|)\,dx$.

12. Denote by a_n the integer closest to $\sqrt{n}$. Find $\sum_{n=1}^{2025}\frac{1}{a_n}$.

13. In $\triangle ABC$, we have $\angle A = 40^\circ, \angle B = \angle C$. Point D on the side AB is such that $\frac{BC}{AD} = \sqrt{3}$. Find $\angle DCB$ in degrees.

14. Find all odd non-prime numbers n such that $(n-1)!$ is not divisible by n^2.

15. Let $y = ax + b$ be the line tangent to the graph of $y = x(x-1)(x-2)(x-4)$ in two points. Find a.

16. Consider the function $f(x) = |4 - 4|x|| - 2$. Find the number of solutions of the equation $f(f(x)) = x$.

17. Find x from the system

$$\frac{x - y\sqrt{x^2 - y^2}}{\sqrt{1 - x^2 + y^2}} = 1, \qquad \frac{y - x\sqrt{x^2 - y^2}}{\sqrt{1 - x^2 + y^2}} = \frac{1}{\sqrt{2}}.$$

18. Positive real numbers x, y satisfy $x^2 + y^2 + \frac{1}{x^2+2x} + \frac{1}{y^2+2y} = 2$. Find $x + y$.

19. Let $a < b < c$ be the roots of $x^3 - 3x + 1 = 0$. Find $a^2 - c$.

20. A cube with side of length 5 is partitioned into 5^3 little cubes with sides of length 1. We choose k little cubes, and draw three lines through the center of each chosen cube parallel to its edges. What is the smallest number k for which we can choose the little cubes in such a way that the lines intersect all 5^3 little cubes?