EF Exam

Texas A&M Math Contest

25 October, 2025

(NOTE: If units are appropriate, please include them in your answer. All answers must be simplified where possible.)

1. A rectangle has perimeter 17 and area 15. Find the length of the diagonal of this rectangle.

2. Let f be a one-to-one differentiable function and let g denote its inverse. Find $h'(3)$ if $h(x) = x^2 g(x)$, $f(2) = 3$, and $f'(2) = 7$.

3. The expression $\left(x + 1 + \dfrac{1}{x}\right)^4$ can be expanded as a sum $\displaystyle\sum_{k=-4}^{4} c_k x^k$, where each c_k is a real number. Find the constant term c_0.

4. Find a real solution x of the equation $\sqrt{x\sqrt{x\sqrt{x}}} = 8\sqrt{2}$.

5. Find the limit of the sequence $a_n = \sin^2(\pi\sqrt{n^2 + n + 1})$.

6. A polynomial $f(x) = x^5 - 2x^4 + ax^2 + bx$, where a and b are unknown coefficients, is divisible by $x^2 - 3x + 2$. Find ab.

7. Evaluate $\displaystyle\lim_{n\to\infty} \left(\frac{4}{3}\sum_{k=1}^{n} \frac{1}{k(k+2)}\right)^n$.

8. Let f be a continuous function such that $\displaystyle\int_{-1}^{1} f(x^2)dx = 2025$. Find $I = \displaystyle\int_{-1}^{1} \frac{f(x^2)}{5^x + 1}dx$.

9. Find the x-coordinate of all the points on the graph of $f(x) = \dfrac{x}{x+1}$ where the tangent line also passes through the point $(1, 2)$.

10. Suppose T is a triangle such that the center of its circumscribed circle lies within the triangle. Let α, β and γ be the angles of T. Find the largest real number c such that the inequality
$$\sin\alpha + \sin\beta + \sin\gamma \geq c$$
is guaranteed for any choice of the triangle T.

11. Determine how many real solutions (x, y) the following system of equations has:

$$\begin{cases} x^2 + y^2 = 18, \\ \sin(x - y) = 0. \end{cases}$$

12. Evaluate $\lim_{n\to\infty} a_n$, where

$$a_n = \frac{\sin\left(1 + \frac{1}{n}\right)}{\sqrt{n^2+1}} + \frac{\sin\left(1 + \frac{2}{n}\right)}{\sqrt{n^2+2}} + \cdots + \frac{\sin\left(1 + \frac{n}{n}\right)}{\sqrt{n^2+n}}.$$

13. Find all solutions x of the equation $\log_3 x + \log_x 27 = \log_3(9x^3) + \log_x(9/x)$.

14. Find the exact value of

$$E = \sum_{k=1}^{8} \sin^6 \frac{(2k-1)\pi}{32}.$$

15. Evaluate the integral $\displaystyle\int_0^{\sqrt{3}} \arcsin\left(\frac{2x}{x^2+1}\right) dx.$

16. A function $f : \mathbb{N} \to \mathbb{N}$ on the set of positive integers is defined recursively as follows: $f(1) = 1$, $f(n) = f(n-1) + 1$ for all odd numbers $n \geq 3$, and $f(n) = f(n/2)$ for all even n. Find $f(2025)$.

17. Consider the trigonometric equation

$$(\sin 2x + \sqrt{3}\cos 2x)^2 - 5 = \cos\left(\frac{\pi}{6} - 2x\right).$$

Find the sum of all solutions of this equation which lie in the interval $[0, 4\pi]$.

18. Evaluate the limit

$$\lim_{n\to\infty} \frac{\sqrt{\binom{n}{2}} + \sqrt{\binom{n+1}{2}} + \cdots + \sqrt{\binom{2n}{2}}}{n^2}.$$

19. Find the exact value of $E = \cot^6\left(\dfrac{\pi}{9}\right) + \cot^6\left(\dfrac{2\pi}{9}\right) + \cot^6\left(\dfrac{4\pi}{9}\right).$

20. Consider the sequences $\{a_n\}$ and $\{b_n\}$ given by

$$a_n = \sum_{k=1}^{n} \frac{3k+1}{k+1} \binom{2k}{k}$$

and $b_n = \sqrt[n]{a_n + 2}$. Find $\lim_{n\to\infty} b_n$.

21. Find all the triples of positive real numbers (x, y, z) that satisfy the equations

$$\frac{4\sqrt{x^2+1}}{x} = \frac{5\sqrt{y^2+1}}{y} = \frac{6\sqrt{z^2+1}}{z},$$
$$x + y + z = xyz.$$